Draft version. First version, September 1989. This version: June 1990
Submitted to the 10th PSCC Conference, August 1990, Graz, Austria

JUANAC: A MODEL FOR COMPUTATION OF SPOT PRICES IN INTERCONNECTED POWER SYSTEMS

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ABSTRACT

Spot prices, i.e., the spatially and temporally varying short-term marginal costs of electricity production with respect to changes in demand, have been frequently advocated as the correct economic signals to be exchanged between the different participants in the electricity market. The computation of spot prices is not a trivial task, since they depend in non obvious ways on the load level, the available generation and transmission equipment and the operating conditions. This paper presents a conceptually straightforward but efficient model for spot price computation, in the context of several interconnected utilities. The model is deterministic and it is based on the DC load flow equations, although it has been modified so that it can include transmission losses. Voltage variations and reactive power are ignored. The dispatch optimization is formulated as a linear programming problem, what makes it easy to include network constraints and scheduled power exchanges. Hydro units are also easily included in the model, with deviations from a scheduled output level being penalized at a prescribed replacement cost. A nine bus, fourteen line system example is fully discussed.The model has been satisfactorily tested on much larger systems.

INTRODUCTION

Spot prices, i.e., the spatially and temporally varying short-term marginal costs of electricity production with respect to changes in demand, have been frequently advocated as the correct economic signals to be exchanged between the different participants in the electricity market [1]. Conceptually they can be used in many ways, ranging from on-line operation of the power system, to 24 hour advance notice to participant consumers and also in the analysis of strategic issues, such as the estimation of the actual cost of wheeling transactions or the assignment of costs of other transmission services in the negotiation of long term power exchange contracts.

The computation of spot prices is not a trivial task, since they depend in non obvious ways on the load level, the available generation and transmission equipment and the operating conditions. In general, spot prices vary from bus to bus, depending on the pattern of transmission losses and on the active transmission limits, i.e., maximum line capacities and bus voltage limits. In case that several independently dispatched utilities are interconnected, the computation of spot prices becomes more involved, since a change in the demand of one bus affects the individual dispatches of all these utilities.

The general mathematical formulation of spot pricing and its areas of application have been extensively described in the technical literature, whith [1] being the most comprehensive treatment of this topic. However little has been said about specific models for practical computations of spot prices. In [2] it is discussed in depth how to modify a security constrained dispatch model to make it suitable for forecasting day-ahead marginal costs for real time pricing and, as in this paper, a linear programming approach is proposed, although no specific model is developed. Besides, the embedded probabilistic features of this method substantially differ from the procedure presented in this paper, where a single deterministic scenario is studied at a time. Recently a computer program named WRATES has been developed, see [3], that not only calculates spot prices in a multiple utility setting, but also computes the rates for wheeling transactions between utilities. Despite these achievements, WRATES still has some limitations, that are partly overcome in the present paper: the iterative solution procedure in WRATES is slow and sometimes presents convergence problems; the model does not include hydro plants; only one hard constraint for line overloading can be imposed; so far only models of reduced dimension have been handled; finally, the model is deterministic.

This paper presents a conceptually straightforward but efficient model for spot price computation, in the context of several interconnected utilities. The model is deterministic and, as in WRATES, it is based on the DC load flow equations, although it has been modified so that it can include transmission losses. Voltage variations and reactive power are ignored. The dispatch optimization is formulated as a linear programming problem, what makes it easy to include network constraints and scheduled power exchanges. Hydro units are also easily included in the model, with deviations from a scheduled output level being penalized at a prescribed replacement cost.

The LP formulation results in a concise statement of the dispatch problem, with the spot prices naturally resulting from simple combinations of the dual variables. Quadratic programming is another alternative, since spot prices can also be derived from the resulting Lagrange multipliers as it is reported in [11].

It is important to note that the deterministic model presented in this paper is a spin-off of a larger project sponsored by Hidroeléctrica Española, a spanish electric utility, where the goal is to compute reliability indices and the expected values of production costs and spot prices for a large composite generation and transmission system. Monte Carlo simulation will be used to generate the different scenarios that will be examined with a dispatch model similar to the one in this paper. Ideas from similar studies, see [4 to 8], can be found in our approach.

In the next section the basic features of the adopted model are qualitatively described and then mathematically stated. Section 3 presents the linear programming solution that has been selected and the way in which spot prices are derived from it. The application to a sample case is discussed in section 4. Finally, section 5 presents the conclusions and offers suggestions for further work. The mathematical derivation of the extended DC network model used in this paper is presented in an appendix.

2. THE MODEL

2.1. Glossary of terms

Definition of indices

- i: Bus number $i = 1 \ldots I$
- ng: Thermal generation unit number at bus i $ng = 1 \ldots NG_i$.
- nd: Unserved energy class at bus i $nd = 1 \ldots ND_i$
- u: Utility number, $u = 1 \ldots U$

Generation and load related terms

gt_i^{ng}, Cgt_i^{ng}:	Thermal power output of unit ng at bus and its cost per MW.
$\overline{gt}_i^{ng}$, $\underline{gt}_i^{ng}$:	Max. and min. output limits for gt_i^{ng}.
gt_i:	Total thermal power output at bus i.
r_i^{nd}, Cr_i^{nd}:	Unserved energy of class nd at bus i, and its cost per MW.
r_i, Cr_i:	Total unserved energy at bus i and its cost.
$sghp_i$:	Scheduled hydro power output at bus i.
ghr_{Mi}:	Maximum hydro power output available above $sghp_i$ at bus i.
gh_i:	Total hydro power output at bus i.
ghp_i:	Scheduled component of gh_i.
ghr_i, $Cghr_i$:	Excess above scheduled hydro power at bus i and its cost penalty per MW.
Cgh_i:	Total cost of hydro power output at bus i.
g_i, d_i, r_i:	Total power output, load demand and unserved energy at bus i.
(g), (d), (r):	Vectors of power outputs, load demands and unserved energies at all buses except for the slack bus s.

Network related terms

θ_i:	Voltage angle at node i.
y_{ij}, Y:	line i,j admittance,network admittance matrix
$G_{i,j}$:	Real component of the admittance of line i,j.
$l_{i,j}$:	Ohmic losses in line i,j.
$l'_{i,j}$, $TLFP_{i,j}$:	Coefficients of the linear approximation to the loss equation.
$z_{i,j}$, $\bar{z}_{i,j}$:	Power flow and capacity limit of line i,j.
$z_{i,j}^e$, $z_{i,j}^s$:	Power flow entering and leaving line i,j.
$NOPF_u$:	Net scheduled output power flow from utility u
rc_u, Cf_u, Ctf_u:	Deviation from the exchange requirements of utility u, associated cost penalty per MW and resulting total cost for the utility.

2.2. Basic features of the model

In this section the characteristics of each one of the components of the model, i.e.,the generation, the load, the network and the utility dispatch coordination, will be described. Then the global formulation of the problem will be presented.

2.2.1 The load and the thermal generation models

The model physically consists of a network where its nodes have an associated load demand and a set of generation units. Although a large number of nodes may be considered, the user may decide to aggregate several of these nodes into areas (the aggregation of the lines of the network will be discussed later) in order to reduce the computational requirements or because a simplified version of the system is preferred. An area is treated as a normal node with an equivalent load that is the sum of the loads of the individual nodes and with a set of generating units.

The cost of generation of the individual units is modeled as a linear function of the power output, whose coefficients must be provided by the user. Every generating unit has associated with it a value of the technical minimum (i.e., minimum output that the unit can provide when it is in operation) and also of the maximum power output. These values are strictly observed, even in the node that is later chosen as the slack bus, since otherways the values of the spot prices may be distorted and the solution of the dispatch may not make physical sense.

The entire list of units provided by the user is assumed to be in operation and connected to the grid, i.e., generating at least the technical minimum. As it was mentioned before, the model described in this paper only computes the optimal dispatch and the spot prices for a given situation of availability and commitment of the generating units and the network. If desired, other higher level models may be used to determine which components are available and also to program their commitment to operate.

Since only the cost and the power output limitations differenciate the generating units within the same node, it is straightforward to build a piece-wise linear function that represents the combined cost of generation at a given node in terms of the total thermal generation output of the node. This function takes into consideration the priority of the technical minima and arranges the units by economic merit order.

$$gt_i = \sum_{ng=1}^{NG_i} gt_i^{ng} \qquad \underline{gt}_i^{ng} \le gt_i^{ng} \le \overline{gt}_i^{ng} \qquad (1.a)$$

$$Cgt_i = \sum_{ng=1}^{NG_i} Cgt_i^{ng} \; gt_i^{ng} \qquad (1.b)$$

The amount of load r_i that may be left unserved at the i-th node is assigned a cost, which is typically high compared

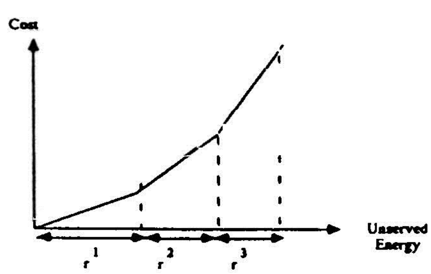

Figure 1 Unserved energy cost function

to generation costs, and that must be provided by the user in the form of a piece-wise linear function, see Fig. 1.The different slopes in this cost function may be used to capture the differences between the failure costs perceived by various types of consumers that are connected to the same node or to represent diverse kinds of contracts.

Consequently the unserved energy at the i-th node and its cost will be given by the expressions:

$$r_i = \sum_{nd=1}^{ND_i} r_i^{nd} \qquad Cr_i = \sum_{nd=1}^{ND_i} Cr_i^{nd}\ r_i^{nd} \qquad (2)$$

2.2.2. The hydroelectric generation model

A separate treatment of the hydroelectric generation is justified because of several reasons, which include the relevance of this technology in the spanish power system. The possibility of storage, the zero variable costs and the existence of a limited amount of total available energy, require that higher level functions (i.e., mid-term hydro-thermal optimization) must be used to determine the target schedule of generation of the hydro units for the dispatch level, based on the unit availabilities, weather and load forecasts, reservoir levels and the many constraints regarding water use and operation of the power system. Here it will be assumed that a previous function, such as the probabilistic module referred to previously, has already specified the amount of hydro generation that is desirable to spend during the time interval considered in this model.

The procedure adopted in this model to deal with the eventual difference between the prespecified target hydro production at the i-th node $sghp_i$, set by the higher order function, and the actual production ghi decided by this model, has been already used in [4]. It simply consits of considering as admissible any value of the actual hydro production ghi comprised between zero and a specified maximum hydro generation $sghp_i + ghr_{Mi}$, which depends on the hydro units and the amount of water that is currently available. Then gh_i is artifically divided into two components

$$gh_i = ghp_i + ghr_i \qquad (3)$$

with $\quad 0 \le ghp_i \le sghp_i \quad , \quad 0 \le ghr_i \le ghr_{Mi}$

where positive values of ghr_i are penalized with a per unit cost of $Cghr_i$, since it is understood that the dispatch will recur to this extra hydro generation only in emergency cases where only very expensive thermal generation is available, or even when there is risk of nonserved energy. The value of this penalty can be approximately estimated as the cost of replacement (at a later period) of the extra water that is used now. On the other hand, there is no penalty associated to having values of ghp_i under the programmed target value $sghp_i$, since this increases the expected water reserves and it will happen only in extraordinary occasions (e.g., when hydro generation at a node is restricted by the available network capacity). The LP optimization mechanism will make sure that whenever ghp_i is under its target value, the component ghr_i is equal to zero. Therefore the cost associated with hydro generation at the i-th node is simply:

$$Cgh_i = Cghr_i \cdot ghr_i \qquad (4)$$

2.2.3. The network model

Coherently with the possibility of aggregation of nodes into areas, lines may also be grouped into corridors. As with generating units, lines keep their own identity so it is easy to create different availability scenarios by simply modifying the data base.

Capacity limits of lines are modeled as hard constraints and they are strictly observed. This is an important feature of the model as it directly affects spot price values.

The network model used in this paper is based on the DC load flow approximation, which has been extended to account for losses. A detailed derivation of this model can be found in the apprendix. The selected method represents a reasonable trade-off between a full AC load flow and the standard DC approximation, since transmission losses are important for the spatial diversity of spot prices. However voltage considerations are ignored.

Losses in a line are modeled as fictitious loads connected at each end node of the line. Each fictitious load represents half of the total losses of the line. These losses are calculated from an approximated equation, see [10]:

$$l_{ij} = 2\, G_{i\text{-}j}\, (\, 1 - \cos(\theta_i - \theta_j)) \qquad (5)$$

where i and j are the nodes at the end buses of the lines.

Since (5) is the only nonlinear expression in the model, it has been decided to use an iterative procedure to solve the resulting optimization problem, so that only the linearized version of (5) about the current approximation to the actual operating point is used in the model.

The complete DC load flow equations, including the linear approximation to the losses, is represented by the following node balance equations and total balance equations (see the appendix):

$$(Y + \frac{1}{2}TLFP)\,(\theta) + (g) + (r) = (d) + \frac{1}{2}(L') \qquad (6)$$

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i + \sum_{i,j}^{I,J} \frac{1}{2}\ TLF_{i,j}\,(\theta_i - \theta_j) = \sum_{i=1}^{I} d_i + \sum_{i,j}^{I,J} l'_{i,j} \qquad (7)$$

where $l'_{ij} + TLFP_{ij}(\theta_i - \theta_j)$ represents the linear approximation to the losses of the line i,j about an operating point defined by $\theta_i - \theta_j$, see Fig. A-2.

In the first pass of the iterative procedure losses are ignored, since all angles (θ) are initially set to zero and (6,7) reduce to the classical DC approximation. In succesive iterations the last values obtained for (θ) are used to update the linear approximations to the loss equations, by modifying the coefficients (L') and (TLFP).

In the realistic cases that have been run, 2-3 iterations were typically needed to converge to the actual solution within satisfactory accuracy.

The linearized model (6,7) happens to be very convenient in the derivation of spot prices, as it will be explained in section 3.2.

2.2.4. Utility dispatch configuration

The system to be studied may consist of a single utility or of several utilities with prespecified power exchanges between them. In both cases centralized dispatch is assumed. The possibility of having independently dispatched entities, see [2], is being introduced in a revised version of this model.

When several utitilies exist, each line must be completely assigned to one of them, in order to avoid ambiguites with the amounts of power being exchanged. If it is desired to model a line as belonging to two utilities, it may be done by adding a new fictitious node somewhere in the middle of the line.

Power exhange agreements between utilities can be easily modeled by adding constraints to the network model, that require that the net flows in or out from each utility must have some user-specified values. The model tries to observe the power exchange contracts, and it assigns a penalty per MW of deviation when the scheduled exchanges are not exactly met. The equation to compute the deviation takes the following form for the u-th utility:

$$\sum_{i,j\varepsilon E_u} z^e_{i,j} + \sum_{i,j\varepsilon S_u} z^s_{i,j} + rc_u = NOPF_u \qquad (8)$$

where E_u = {Interconnection lines not owned by utility u}
S_u = {Interconnection lines owned by utility u}

where rc_u is the deviation in MW of utility u in meeting its exchange requirements. The flows $z^e_{i,j}$ and $z^s_{i,j}$ can be easily written in terms of the line flows $z_{i,j}$ and losses $l_{i,j}$, and are needed to precisely write the amount of real power that enters or leaves a certain node, see the appendix.

The dispatch function tries to minimize the total operation costs for the entire system. These costs include the costs of thermal generation, the costs of unserved energy, the costs of deviation from the scheduled hydro program and the costs of failing to meet the power exchange contracts. This minimization is subject to hard constraints concerning line capacity limits and maximum and minimum generation output limits. The minimization algorithm must be called several times due to the iterative procedure that has been employed to deal with the nonlinearity in the loss model.

Note that this model only tries to dispatch the units already connected to the network in order to minimize the total operation cost of meeting the current demand. It is supposed that higher order functions have taken care of scheduling the adequate number of units for operation so that operating reserves are properly accounted for.

2.3. The global problem formulation

The global dispatch problem can be formulated as the following LP optimization problem:

$$\text{Minimize } Z = \sum_{i=1}^{I} Cgt_i + \sum_{i=1}^{I} Cgh_i + \sum_{i=1}^{I} Cr_i + \sum_{u=1}^{U} Ctf_u \qquad (9.a)$$

where,

$$Cgh_i = Cghr_i \cdot ghr_i \qquad Ctf_u = Cf_u \cdot rc_u \qquad (9.b)$$

$$gt_i = \sum_{ng=1}^{NG_i} Cgt_i^{ng}\, gt_i^{ng} \qquad Cr_i = \sum_{nd=1}^{ND_i} Cr_i^{nd} \cdot r_i^{nd} \qquad (9.c)$$

subject to the following equality and inequality constraints:

(i) Network load flow equations:

$$(Y + \frac{1}{2}TLFP)(\theta) + (g) + (r) = (d) + \frac{1}{2}(L) \qquad (10)$$

(ii) Total balance of power equation:

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i + \sum_{i,j}^{I,J} TLFP_{i,j}(\theta_i - \theta_j) = \sum_{i=1}^{I} d_i + \sum_{i,j}^{I,J} l'_{i,j} \qquad (11)$$

(iii) Line capacity limits

$$-\bar{z}_{i,j} \le z_{i,j} \le \bar{z}_{i,j} \qquad i,j = 1 \quad I,J \qquad (12)$$

(iv) Output limits of the generation units

$$\underline{gt}_i^{nd} \le gt_i^{nd} \le \bar{gt}_i^{nd} \qquad = 1 \qquad (13.a)$$

$$0 \le ghp_i \le sghp_i \qquad 0 \le ghr_i \le ghr_{Mi} \qquad i = 1 \qquad (13.b)$$

(v) Physical limit for the unserved energy at each node·

$$0 \le r_i \le d_i \qquad = 1 \qquad (14)$$

(vi) Satisfaction of scheduled power exchanges:

$$\sum_{i,j\varepsilon E_u} z^e_{i,j} + \sum_{i,j\varepsilon S_u} z^s_{i,j} + rc_u = NOPF_u \; , u = 1 \quad U \qquad (15)$$

3. THE SOLUTION METHOD

3. The optimization algorithm

The optimization problem that was presented in the last section can be summarily restated as the minimization of a linear objective function (9) subject to a set of linear constraints (10 to 15).

$$\text{Min}_x \; Z(x,y) \qquad (16)$$

$$\text{subject to } A_1 x + A_2 y \ge b \qquad (17.a)$$

$$x_{lower} \le x \le x_{upper} \qquad (17.b)$$

where x is the set of thermal power outputs gt_i's, the hydro power output components ghp_i's and ghr_i's, the unserved load demands r_i's, the contract power exchange deviations rc_u's, and where y is the vector of voltage angles θ_i's.

The restrictions (17) have been enumerated in the last section. Although they were not explicity writen in terms of x and y, it is immediate to translate them into the form (17). This format corresponds to a linear programming optimization problem, that can be accepted by well proven optimization packages such as MINOS [9].

Fig. 2 represents the organization of the iterative process that has been employed to deal with the problem of the nonlinearity in the loss equation. The process is initiated by ignoring the losses in the network model, so a first approximation to the actual solution is provided by the optimization module. The voltage angles in this solution are used to update the coefficients $TLFP_{i,j}$ and $l'_{i,j}$ in the network equations, see (A.11 to A.13). Then the optimization module can be applied again and the new solution is tested for convergence, for instance by checking its difference with the preceding solution.

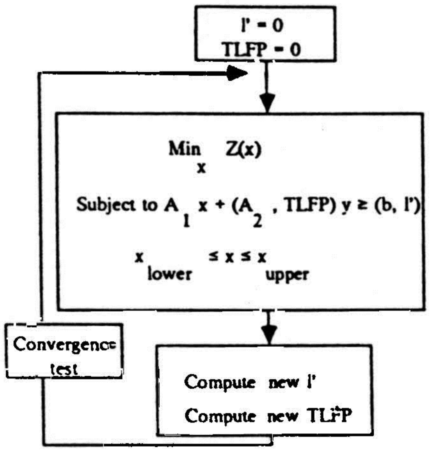

Figure 2 Organization of the iterative process

3.2. Spot prices computation

Spot prices, i.e., the spatially and temporally varying short-term marginal costs of electricity production with respect to changes in demand, are mathematically formulated as:

$$\rho_i = \frac{\partial Z}{\partial d_i} \quad (18)$$

Its computation is not a trivial task since the derivative must be computed while observing the output generations and line capacity limits, the scheduled power exchanges and the network equations including losses. Fortunately it will shown next that spot prices can be directly obtained as simple combinations of dual variables of the solution to the LP problem that was formulated in section 2.3.

Considering the problem formulation (16, 17), the dual variable Vd_j of any active restriction with a generic format $\sum_{i=1}^{n} a_{j,i}\, x_i = b_j$, represents the change in the optimal value of the objetive function Z caused by a unit change in b_j, i.e., $Vd_j = \frac{\partial Z}{\partial b_j}$, where all the remaining restrictions must be respected. It is immediate to realize that the load demand d_m at the mth node always appears in the detailed formulation of section 2.3 as part of the right hand side of some constraints. This suggests a close relationship between the dual variables of these constraints and the spot prices. However the exact derivation of spot prices from dual variables requires a careful analysis since the correspondence is not immediate.

An important practical point is that the LP algorithm ignores that d_m appears in the right hand side of several constraints, since the algorithm only sees the numerical values of these right hand sides. Therefore, in order to compute $\partial Z/\partial d_m$ it will be necessary to separately study the impact on Z of a unit change of d_m in the right hand side of every constraint where d_m appears.

Inspection of the formulation in section 2.3 shows that d_m appears in the power balance equation and in the upper limit of the unserved energy at the m-th node. Rewriting these three equations in a way that makes explicit that the sensitivities will be independently computed for changes in d_m in each one of them:

$$- \sum_{k \in K_m} (\gamma_{m,k} + \frac{1}{2} TLFP_{m,k})(\theta_m - \theta_k) + g_m + r_m = d_{m1} + \sum_{k \in K_m} \frac{1}{2} l'_{m,k} \quad (19)$$

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i + \sum_{\substack{i,j \\ i \neq m}}^{I,J} TLFP_{i,j}(\theta_i - \theta_j) + \sum_{\substack{m,k \\ k \in K_m}} TLFP_{m,k}(\theta_m - \theta_k) = \sum_{\substack{i,j \\ i \neq m}}^{I} d_i + d_{m2} + \sum_{\substack{i,j \\ i \neq m}}^{I,J} l'_{i,j} + \sum_{\substack{m,k \\ k \in K_m}} l'_{m,k} \quad (20)$$

$$0 \leq r_m \leq d_{m3} \quad (21)$$

then the spot price can be calculated as

$$\rho_m \triangleq \frac{\partial Z}{\partial d_m} = \frac{\partial Z}{\partial d_{m1}} + \frac{\partial Z}{\partial d_{m2}} + \frac{\partial Z}{\partial d_{m3}} \quad (22)$$

Equation (19) with its dual variable Vd_1 is examined first.The important point to recognize is that a dual variable is a first order (i.e., linear) sensitivity and therefore that the linear approximation to the nonlinear loss equation does not introduce any error, subject to the condition that the linearization is performed about the correct operating point. Therefore the dual variable Vd_1 of the linearized constraint (19) is exactly what is needed and

$$\frac{\partial Z}{\partial d_{m1}} = Vd_1 \quad (23)$$

The same rationale leads to the conclusion that

$$\frac{\partial Z}{\partial d_{m2}} = Vd_2 \quad (24)$$

since, when d_{m2} changes, all the remaining d_i's are supposed to remain constant.

With respect to (21) it must be noticed that the inequality $r_m \leq d_{m3}$ may be active or not, with Vd_3 being zero in the later case. In any case it is always true that

$$\frac{\partial Z}{\partial d_{m3}} = Vd_3 \quad (25)$$

therefore yielding the final expression for the spot price:

$$\rho_m = Vd_1 + Vd_2 + Vd_3 \quad (26)$$

3.3. Computer implementation

Here the LP optimization problem has been solved using the package MINOS [9], which complies very well with the requirements of this application, namely a large sparse system of equations and the need for efficient computation of the dual variables.

MINOS is a large-scale optimization program for the solution of sparse linear and nonlinear systems. The objective function and the constraints may be linear or nonlinear. Stable numerical methods are employed troughout. Features of MINOS include a new basis package (for maintaining sparse LU factors of the basis matrix), automatic scaling of all constraints and automatic estimation of some or all the gradients. Upper and lower bounds on the variables are handled efficiently. File formats for constraint and basis data are compatible with industry MPS format. The specified names for the constraints and the variables, as well as their bounds and initial values are supplied in a file defined in a standad format which is read by MINOS. Because MINOS is often called, the efficiency of the process can be enhanced by substituting the standard file to be read by MINOS with a vector file so that the costly time computation of opening and reading a file is eliminated.

MINOS automatically performs the computation of dual variables of the optimization problem (16,17), i.e. shadow prices of the constraints (10 to 15). The values corresponding to the constraints (10),(11) and (14) are the only ones strictly needed for spot price computation; the remaining ones provide information that may be useful for other purposes.

The complete package, which has been named JUANAC, comprises, besides the MINOS code, a module to compute the losses and the coefficients of the linearized expression, a spot price calculation module and the module that controls the iterative process. Fig. 3 shows a simplified flowchart of JUANAC.

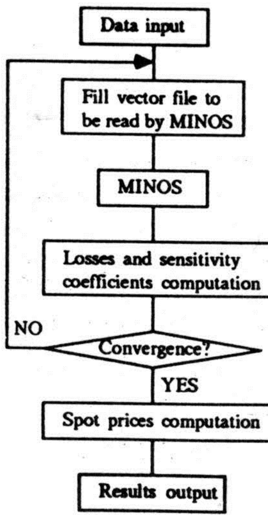

Figure 3 Flowchart of JUANAC

JUANAC has been implemented in aDIGITAL computer, model VAX station 2000, with 4 Mb of main memory. The program has been written in Fortran 77. The case study has 34 variables, 49 constraints and it takes approximately 14 seconds of CPU time to run.

4. CASE STUDY

Two simple examples demonstrate the use of the program. First the program has been applied to a nine bus, fourteen line system where both the generation and the line capacity limits have been exagerately increased in order to better show the spot price dependence on losses. Fig. 4. depicts the system configuration and Table 1 and 2 summarize the generation/load and the network data respectively.

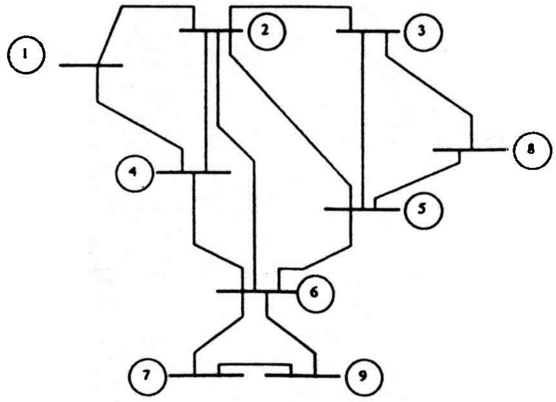

Figure 4 System configuration

Table 1 Generation/Load Data

Bus	Hydro generation (MW)	Thermal generation						Load	
		Unit 1		Unit 2		Unit 3		Demand	Unserved
		(MW)	(Cost)	(MW)	(Cost)	(MW)	(Cost)	(MW)	(Cost)
1	300	75	65	125	70	100	75	1	15000
2	-	100	59	50	67	50	74	240	15000
3	160	100	61	50	76	50	80	40	15000
4	-							160	15000
5	-							240	15000
6	150							80	15000
7	-							100	15000
8	100							15	15000
9	-							100	15000

Table 2 Line Data

LINE From	To	Reactance (P.U.)	Resistance (P.U.)	Capacity Limit (MW)
1	2	0'2913	0'0777	500
1	4	0'2041	0'0544	500
2	3	0'1695	0'0424	500
2	4	0'4	0'1	500
2	5	0'2	0'05	500
2	6	0'4	0'1	500
3	5	0'099	0'0248	500
3	8	0'4	0'1	500
4	6	0'6	0'15	500
5	6	0'2	0'05	500
5	8	0'4	0'1	500
6	7	0'6	0'15	500
6	9	0'2	0'05	500
7	9	0'2	0'05	500

Table 3 Results

Bus	Generation output (MW)		Spot Price	Unserved energy	Line	Power Flow (MW)
	Thermal	Hydro				
1	32'79	300	65	-	1 -> 2	
2	200		76'79	-	1 -> 4	
3	100	160	73'79	-	2 -> 3	
4			78'35	-	2 -> 4	
5			80'61	-	2 -> 5	
6		150	85'05	-	2 -> 6	
7	-	-	101'09	-	3 -> 5	
8		100	71'45	-	3 -> 8	
9			97'34	-	4 -> 6	
					5 -> 6	
					5 -> 8	
					6 -> 7	
					6 -> 9	
					7 -> 9	

Dispatch results,including spot prices,are shown in Table 3 Differences between them (because of the losses) are significant.

In the second place a six bus, eight line system is studied (Fig. 5.). Here the data, which is summarized in Table 4 has been prepared so that there is a bus with unserved energy and a line capacity limit is active. Results are presented in Table 5. It must be noticed that the spot price at the bus where there is unserved energy coincides with the cost per MW of unserved energy and that the spot price at the bus where there is still hydroelectric generation available is zero, also as expected.

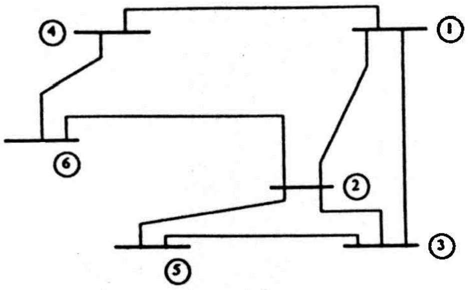

Figure 5 System configuration

Table 4 Generation Load Data

Bus	Hydro generation (MW)	Thermal generation						Load	
		Unit 1		Unit 2		Unit 3		Demand	Unserved energy
		(MW)	(Cost)	(MW)	(Cost)	(MW)	(Cost)	(MW)	(Cost)
1	150							80	15000
2								240	15000
3								160	15000
4								240	15000
5	300	50	65	100	70	50	75		15000
6	160	50	61	100	76	50	80	40	15000

Table 5 Results

Bus	Generation output (MW)		Spot Price	Unserved energy (MW)	Line	Capacity Limit (MW)	Power Flow (MW)
	Thermal	Hydro					
					1 -> 2	100	
1		150	9680'7		1 -> 3	80	
2			8165'3		1 -> 4	100	
3			15000	13'02	2 -> 3	80	
4			9296'9		2 -> 5	150	
5		273'09	0		2 -> 6	150	
	200	160	8101'6		3 -> 5	150	
					4 -> 6	250	

CONCLUSIONS

This paper has presented an efficient model for spot pricing computation of a power system that is completely specified in terms of available components in operation (lines and generating units connected to the grid), hydroelectric power target schedule, demand and power exchanges between utilities. This model can be used as a stand-alone program that allows one to examine different power system scenarios that are input by the user. Work is under way to use it as the operation module of a probabilistic package for reliability analysis and production cost calculation of composite generation/transmission systems, where the scenarios will be generated by Monte Carlo techniques.

The adopted formulation of the model and the solution algorithm, which is based on the MINOS code, afford a realistic consideration of the most relevant features in the operation of a power system and lead themselves to the use of a simple procedure for spot price computation. On-going extensions to the model include the consideration of independently dispatched entities (i.e., utilities or independent producers) and the study of alternative formulations of the loss model.

6. ACKNOWLEDGEMENTS.

Mr. Juan Manuel Rodríguez García and Mr. Ignacio Lapuente Bujía made important technical contributions in the initial stages of the project. The first author of this paper has been supported during the realization of this work by a research grant from Hidroeléctrica Española, S.A. . This support is gratefully acknowledged.

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Appendix A. DERIVATION OF THE NETWORK EQUATIONS

The well-known DC network model (see [10] for instance) represents the line flow between the i-th and j-th nodes as:

$$z_{i,j} = \gamma_{i,j} \, (\theta_i - \theta_j) \qquad \text{(A.1)}$$

and the first Kirchhoff's law at the i-th node as:

$$g_i - (d_i - r_i) - \sum_{k \in K_i} z_{i,k} = 0 \qquad \text{(A.2)}$$

where K_i is the set of all nodes adjacent to the i-th node. From (A.1) and (A.2)

$$\sum_{k \in K_i} \gamma_{i,k} \, (\theta_i - \theta_k) + g_i + r_i = d_i \qquad \text{(A.3)}$$

which can be written in matrix format for all nodes (except for one of them, the slack node S, that is arbitrarily chosen and where θ_S is set to a reference value, typically zero. This is a consequence of the fact that there is a redundant equation if the first Kirchhoff's law is applied to all the nodes in a network):

$$(Y).(\theta) + (g) + (r) = (d) \qquad \text{(A.4)}$$

The set of equations (A4) allows one to compute (θ) in a network described by (Y) when (g), (d) and (r) are given. (A4) can be used in a diversity of contexts, where (g), (d) and (r) may or not be externally specified. To ensure the consistency of the values of (g), (d) and (r), the following conservation law has to be included

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i = \sum_{i=1}^{I} d_i \qquad \text{(A.5)}$$

This DC formulation implicitly assumes that network losses can be ignored, since the line flow (A.1) is the same at both ends of a line. A procedure that has been frequently used to include losses in a DC network model, see for instance [10], is to include the losses l_{ij} of each line (i,j,) as fictitious loads at the ends of the line, see Fig. A.1, therefore making the flows at the ends of the line to differ by an amount equal to the losses of the line. Somewhat arbitrarily one half of the total loss of the line is assigned to each end of it. It can be easily shown from Fig. A.1 that

$$z^e_{i,j} = z_{i,j} - \frac{1}{2} l_{i,j} \quad , \quad z^s_{i,j} = z_{i,j} + \frac{1}{2} l_{i,j} \qquad \text{(A.6)}$$

and therefore $\quad z^s_{i,j} - z^e_{i,j} = l_{i,j} \qquad$ (A.7)

as intended. However the detailed notation $z^e_{i,j}$ and $z^s_{i,j}$ is not needed, since it suffices with simply extending (A.3) as

$$\sum_{k \in K_i} \gamma_{i,k} \, (\theta_i - \theta_k) + g_i + r_i = d_i + \frac{1}{2} \sum_{k \in K_i} l_{i,j} \qquad \text{(A.8)}$$

so that (A.4.) and (A.5) become

$$-(Y)\,(\theta) + (g) + (r) = (d) + \frac{1}{2}(L) \qquad \text{(A.9)}$$

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i = \sum_{i=1}^{I} d_i + \sum_{i,j}^{I,J} l_{i,j} \qquad \text{(A.10)}$$

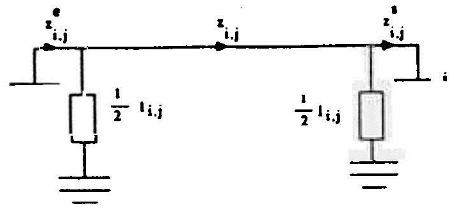

Fig. A.1. Fictitious loads account for the line losses

An expression to model the loss $l_{i,j}$ in a line is now needed. A good approximation is given in [10] (See Fig. A.2.):

$$l_{i,j} = 2\,G_{ij}\,(1 - \cos(\theta_i - \theta_j)) \qquad \text{(A.11)}$$

where G_{ij} is the real component of the line admittance.

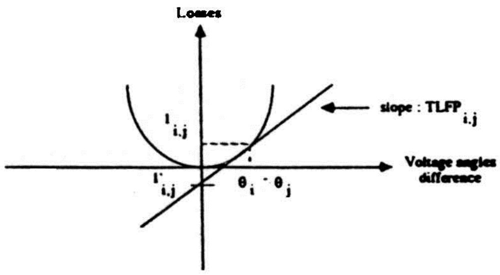

Fig. A.2. The loss model of a line

An obvious difficulty with equation (A.11) is that it spoils the previous linearity of the system of equations (A.9). To cope with this problem, here it has been decided to follow an iterative procedure where the current best values of the bus angles (θ) are used to write a linear approximation to (A.11) about this current operating point (see Fig. A.2):

$$l_{i,j} = l'_{i,j} + TLFP_{i,j}\,(\theta_i - \theta_j) \qquad \text{(A.12)}$$

where,

$$l'_{ij} = 2\,G_{ij}\,(1 - \cos(\theta_i - \theta_j)) - TLFP_{i,j}\,(\theta_i - \theta_j) \qquad \text{(A.13)}$$

$$TLFP_{i,j} = \frac{\partial l_{i,j}}{\partial \theta_i} = -\frac{\partial l_{i,j}}{\partial \theta_j} = -TLFP_{j,i} = 2G_{ij}\,\mathrm{sen}\,(\theta_i - \theta_j) \qquad \text{(A.14)}$$

The linearized version of (A.8) is now:

$$\sum_{k \in K_i} (\gamma_{i,k} + \frac{1}{2} TLFP_{i,k})(\theta_i - \theta_k) + g_i + r_i = d_i + \sum_{k \in K_i} \frac{1}{2} l'_{i,k} \qquad \text{(A.15)}$$

and in matrix format

$$-(Y + \frac{1}{2} TLFP)\,(\theta) + (g) + (r) = (d) + \frac{1}{2}(L) \qquad \text{(A.16)}$$

The linearized version of (A.10) becomes:

$$\sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i + \sum_{i,j}^{I,J} TLFP_{i,j}\,(\theta_i - \theta_j) = \sum_{i=1}^{I} d_i + \sum_{i,j}^{I,J} l'_{ij} \qquad \text{(A.17)}$$